

DISCHARGE COEFFICIENTS OF NOZZLES AND OF THEIR COMBINATIONS IN FORWARD AND REVERSE FLOWS

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It is known that during fluid or gas outflow through an orifice in a thin wall or through a short nozzle ($L < 2-3d$) the minimum cross-sectional area of the jet is always smaller than that of the orifice, because of the perpendicular component of the flow velocity and the nonuniform velocity field. Therefore, the actual fluid- or gas-mass flows are always smaller than those calculated from the cross-sectional area of the orifice (their ratio is known as the discharge coefficient μ). Problems of discharge were first studied theoretically by Saint-Venant, Boussinesq, and Kirchhoff. In Russia N. E. Zhukovskii was first to derive formulas for the contraction coefficient of a liquid jet for discharge through a narrow slot [1]. To date, many investigations have been performed in this line, and the results are best presented in [2, 3].

This paper is devoted to the search for nozzles and their combinations such that the difference between discharge coefficients for the forward μ_1 and reverse μ_2 directions is maximum. We studied the range of fairly large supercritical pressure drops in nozzles when the outflowing jet was supersonic.

Experimental Setup. A schematic diagram of the experimental setup is shown in Fig. 1. A nozzle or a combination of nozzles 1 in a 16-mm-diameter channel 2 were placed in a vessel 3 of volume $V = 1.6$ liters. The vessel was filled with air (to a pressure of 2.0 MPa) which discharged into the atmosphere through the nozzle upon opening of a quick-acting valve 4. The cross-sectional areas of the valve and of the short pipelines were larger than those of the nozzles by a factor of 10 and greater. The air-pressure drop in the vessel was measured by the semiconducting pressure gauge 5 connected in a bridge circuit and recorded by an oscilloscope.

Figure 2 shows the tested nozzle shapes: a) is a cylindrical nozzle (for reverse flow it is known as the Borda nozzle) with $d = 5.1$ mm and $L = 15$ mm; $d = 3.4$ mm and $L = 9$ mm; b) is a conic nozzle with an apex angle of 45° and $d = 3.4$ mm; c) is a profile nozzle with a smooth entry with diameters $d = 5.1, 4.65, 4.45, 3.55,$ and 2.9 mm; d) is a cylindrical nozzle with a flattened exit and a smooth entry; e) is a thin orifice plate with sharp rectangular edges; and f) is a thin orifice plate nozzle with a conic orifice and a sharpened edge.

Nozzles with both sharp and blunt edges and also their combinations were tested. To avoid air outflow past the orifices, the outer contours of the nozzle edges were sealed with Teflon rings.

Experimental Procedure. The volume of the vessel and the cross-sectional area of the tested nozzles were chosen so that the time t of supersonic air discharge ranged from 0.5 sec to 1.0 sec. Estimations show that, for this time, the characteristic thickness of the thermal air layer near the vessel walls $x \cong \sqrt{at}$ (a is the thermal diffusivity of air) does not exceed 2-3 mm. Since the process of discharge was analyzed for sufficiently high (at least fivefold) pressure drops in nozzles, the value of x decreases at least twofold and did not exceed 1.5% of the vessel diameter. Therefore, we can say with confidence that the air outflow from the vessel was adiabatic. To confirm this conclusion, in some tests the internal surface of the steel vessel was covered with a paper layer, which reduced heat transfer from the walls during air discharge. This did not change the nature of the discharge within the pressure-measurement error.

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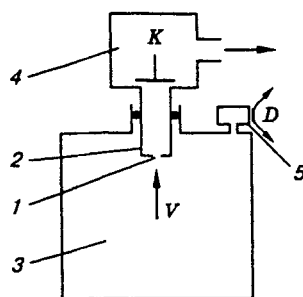


Fig. 1

Solution of the problem of supersonic adiabatic gas discharge from a finite volume gives the following well-known relations:

$$p/p_0 = (1 + ct)^{-2\gamma/(\gamma-1)}, \quad \rho/\rho_0 = (1 + ct)^{-2/(\gamma-1)}, \quad G/G_0 = (1 + ct)^{-(\gamma+1)/(\gamma-1)}.$$

Here p , ρ , and G are the absolute pressure, density, and mass flow of the gas, respectively (the subscript 0 refers to the initial state); t is the time from the beginning of the outflow process; γ is the specific heat ratio; and

$$c = \left(\frac{2}{\gamma+1} \right)^{1/(\gamma-1)} \frac{\gamma-1}{2} \sqrt{\frac{2\gamma}{\gamma+1} \frac{p_0}{\rho_0} \frac{S_n \mu}{V}} \quad (1)$$

is a constant which depends on the composition and state of the gas, on the geometry and cross-sectional area S_n of the nozzle, on the volume of the vessel V , and on the gas-flow direction (in terms of the parameter μ);

$$G_0 = V \left. \frac{d\rho}{dt} \right|_{t=0} = V \frac{2}{\gamma-1} c \rho_0.$$

A typical oscillogram of the pressure drop in the vessel with time is given in Fig. 3. Using the known initial pressure p_0 and an arbitrary pressure $p_i > 1.9p_a$, which corresponds to the time t_i in the oscillogram, the following constant was calculated:

$$c = [(p_i/p_0)^{-(\gamma-1)/2\gamma} - 1]/t_i. \quad (2)$$

The discharge coefficient μ was calculated from formula (1). The specific heat ratio γ was assumed to be equal to 1.4. The excess of the pressure p_i over atmospheric pressure p_a ensured supersonic transition at a certain "critical" cross-section of the jet. In this case, there is no need for calculation of flow rates to obtain the discharge coefficient μ , and determination of the constant c does not require conversion of the amplitudes of the curve to the units of pressure. The maximum error of calculation of c depends on the accuracy of measurements of amplitude and time on the oscilloscope screen, which is 5% and 1.5%, respectively, for an S8-13 oscilloscope. In this case, the value of c calculated by formula (2) can differ from its true value by 3.1%. To improve the accuracy, up to ten points on the curve $p(t)$ were calculated and then the mean value of c was determined. As a result, the calculation error was reduced to $\sim 1.0\%$.

Test calculations with a series of oscillograms were also performed. The oscilloscope scale was calibrated for linearity in amplitude and time by calibrated pulses. As the pressure decreased from initial to atmospheric the nonlinearity of the measuring system did not exceed 0.5%, which eliminated systematic measurement errors. To prevent cooling of the gauge during gas discharge, its internal surface was coated with a layer of cup grease, and the time intervals between tests were long enough for the vessel and gauge to reach the room temperature.

The error of calculation of the discharge coefficient μ also depended on the accuracy of measurement of air temperature, which was used to calculate ρ_0 , the volume V , and the cross-sectional area S_n of the nozzle. The temperature was measured by a thermometer with an accuracy of $\pm 1^\circ$, which gave an error of $\sim 0.3\%$. The volume of the vessel was determined both by a measuring instrument and by filling it with water using a

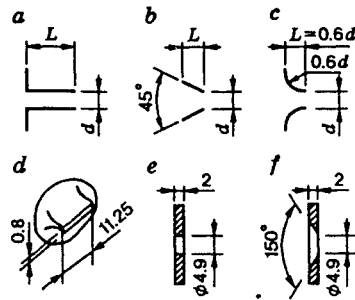


Fig. 2

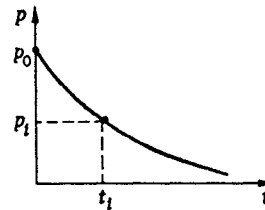


Fig. 3

TABLE 1

Nozzle (Fig. 2)	μ_1	μ_2	μ_2/μ_1
a	0.875	0.766	0.875
b	0.947	0.829	0.876
c	0.983	0.765	0.778
d	0.917	0.825	0.9
e	0.891	0.891	1.0
f	0.9	0.835	0.928

volumetric flask. The error of V measurements did not exceed 0.5%. The quantity S_n was measured using a cone with an apex angle of 3° by a micrometer with an accuracy of 0.01 mm. The error of S_n measurement did not exceed 0.2% for a 5-mm-diameter orifice. Finally, the standard error in the measurements of the discharge coefficient μ did not exceed $\sim 1.2\%$.

Results and Discussion. The results of the tests with single-piece nozzles with sharp edges are listed in Table 1. One can see that the profile with a smooth entry (Fig. 2c) shows the maximum difference between the discharge coefficients μ_1 and μ_2 . For short nozzles of this profile, the coefficient μ_1 depends only slightly on the ratio L/d for supersonic gas discharge, and for Reynolds number $Re = (0.5-1.0) \cdot 10^6$ the difference in values of μ_1 does not exceed 1% as the ratio L/d changes from 1.25 to 2.5, μ_1 growing with a decrease in the ratio L/d [2]. Obviously, the value of μ_2 also depends only slightly on the ratio L/d , since, as can be seen from Table 1, the nozzle discussed and the cylindrical nozzle, whose shape can be obtained by elongating the nozzle with a smooth entry, have very close values of μ_2 . Therefore, the ratio L/d was not varied and was chosen so that nozzles and various combinations of them could be easily made.

Flattening of sharp edges from the side of the entry increases considerably the values of μ . More precisely, chambering of rectangular edges make the values of μ_1 closer to those for a nozzle with a smooth entry (Fig. 2c), and grinding of sharp outward-projecting edges in the direction perpendicular to the axis (Fig. 2a-d, reverse direction) make the values of μ_1 closer to those for the entry with a sharp rectangular edge (Fig. 2a). For example, chambering of the cylindrical-nozzle entry (Fig. 2a) by a radius of 1 mm ($\sim 0.2d$) increases μ_1 to 0.944, and flattening of a sharp projecting edge by a 0.3- and 0.5-mm pad increases μ_2 to 0.873. Note that for subsonic and sonic flows in the reverse direction through the nozzles (Figs. 2a and 2d) theoretical formulas for μ_2 exist which give values $\mu_2 = 0.5-0.63$ depending on the pressure drop [3].

Combinations of two nozzles reduce the discharge coefficients and increase the difference between μ_1 and μ_2 . Again, profiles with a smooth entry show the most considerable difference in μ (Table 2).

The dependence of μ on the distance between nozzles, their relative cross-sectional areas, and number of nozzles placed one after another (Fig. 4) was determined. The values of μ and μ_2/μ_1 as functions of the distance l between two nozzles with smooth entries are shown in Fig. 4a. The minimum of the function μ_2/μ_1

TABLE 2

Nozzle (Fig. 2)	Distance between nozzles l , mm	μ_1	μ_2	μ_2/μ_1
a-a (rounded-off inlet edge)	22	0.85	0.739	0.869
b-b	10	0.82	0.704	0.859
c-c	10	0.92	0.645	0.7
c-a (rounded-off inlet edge in the nozzle a)	10	0.85	0.74	0.87
d-d (flattened edges are parallel)	15	0.825	0.612	0.742
d-d (flattened edges are perpendicular)	15	0.757	0.612	0.808

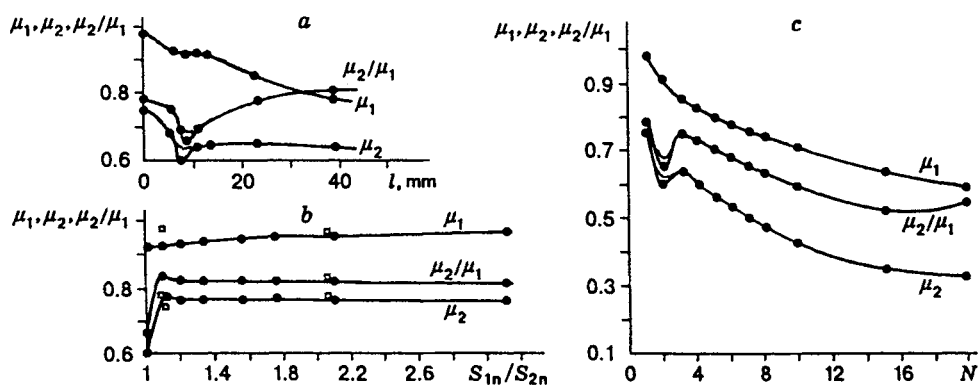


Fig. 4

is located close to $l = 7$ mm ($\cong 1.5d$). In this range of l , the reverse air flow is obviously not stable, and μ_2 takes values from 0.634 to 0.6 in different experiments. The value of μ_1 is rather constant, and, hence, the ratio μ_2/μ_1 varies between 0.689 and 0.652.

Combinations of two nozzles with different diameters show a smaller difference between μ_1 and μ_2 as compared with nozzles with equal diameters placed at the same distance. For two nozzles (with smooth entries) spaced 7 mm apart, the curves of μ_1 , μ_2 , and μ_2/μ_1 versus the ratio of their cross-sectional areas S_{1n}/S_{2n} are shown in Fig. 4b. Nozzles with smooth entries (Fig. 2c) and orifice diameters $d = 5.1, 4.65, 4.45, 3.55,$ and 2.9 mm were used in combinations. The main combinations were composed so that the forward flow enters a nozzle of larger diameter. For comparison the boxes in the graph show the case where the forward flow entered a nozzle of smaller diameter. With a small difference between the cross-sectional areas the latter combination gives a somewhat higher difference in values of μ , but then this advantage is lost. It is seen that even when the ratio of the cross-sectional areas of the nozzles is greater than 1.1 the value of μ approaches the value for a single-piece nozzle of smaller diameter.

Obviously, combinations of a few nozzles placed after one another reduce μ differently for the forward and reverse directions, and, hence, one can obtain any desired ratio μ_2/μ_1 by changing the number of nozzles used. The values of μ_1 , μ_2 , and μ_2/μ_1 as functions of the number N of nozzles with a smooth entry spaced 7 mm apart for $d = 5.1$ mm are shown in Fig. 4c. Combinations of up to 20 nozzles were made. One can see that μ_1 decreases monotonically, and μ_2 and μ_2/μ_1 for two nozzles have a local minimum at which instability of the reverse flow was observed. For a larger number of nozzles, the ratio μ_2/μ_1 does not decrease monotonically but has a minimum of 0.52 in the vicinity of $N = 15$, although μ_1 and μ_2 continue to decrease. In the range $N = 20$, the studied ratio shows a noticeable tendency to increase. Note that the flow in the channel formed by a combination of the same nozzles is always subsonic, and the sonic velocity is attained only in the zone of the final constriction.

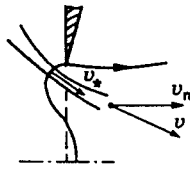


Fig. 5

TABLE 3

p_0 , MPa	μ_1	μ_2
2	0.584	0.328
1	0.562	0.308
0.5	0.551	0.281
0.3	0.487	0.256

Clearly, during liquid discharge or subcritical gas discharge from the vessel through the nozzle orifice, the jet diameter decreases, and, hence, $\mu < 1$ always holds. The same result was obtained by N. E. Zhukovskii. However, there are data in the literature for $\mu > 1$ [3, 4]. This is the case for flows in pipes when the cross-sectional area of the nozzle orifice is greater than 85% of that of the pipe for a shaped-edged nozzle or when it is greater than 20% of the cross-sectional area of the pipe for a nozzle with a smooth entry. Values of $\mu > 1$ are due to an increase in gas discharge on account of the dynamic head of liquid upstream the orifice. Another factor which contributes to the increase in μ is the recovery of the static pressure between the minimum cross-section of the nozzle orifice which is used in calculations and the measuring point downstream the orifice plate. The values of μ depend on the location of the measuring points of the static pressure upstream and downstream the orifice plate, and, hence, these values are meaningless.

For supercritical gas discharge from an orifice, we always have $\mu < 1$. Let us show this. Figure 5 shows the location of the sonic surface near a sharp-edged orifice according to [5] and an elementary flow tube for which

$$\rho_* v_* > \rho v > \rho v_n, \quad (3)$$

where $\rho_* v_*$ is the flow density at the critical cross-section of the pipe; ρv is the current flow density; and ρv_n is the projection of the flow density onto the orifice plane S_n . At a constant flow rate $G = G_* = G_n$, we obtain

$$\int_{S_*} \rho_* v_* dS_* = \mu \int_{S_n} \rho v_n dS_n \quad \text{or} \quad \rho_* v_* S_* = \mu (\rho v_n)_m S_n.$$

On the strength of inequality (3), $S_* < \mu S_n$, but since $S_* > S_n$, we have $\mu < 1$.

During adiabatic air outflow from the vessel, the air temperature decreases from the initial 298 K to ~ 240 K with a pressure drop by a factor of 2.5 and to ~ 180 K with a fivefold pressure drop. Water-vapor condensation occurs which reduces the effective value of the specific heat ratio, while an air temperature rise increases it. However, the value of μ calculated from the curve of the pressure drop were constant with a tenfold pressure drop and for the calculated $\gamma = 1.4$. Actually, the deviation of γ from 1.4 within $\pm 3\%$ does not cause the error of calculation of the value of μ using formula (1) to be greater than the error due to the inaccuracy of the measurements. Therefore, within the measurement error the coefficient μ can be considered constant over the discussed range of pressure drop during adiabatic discharge.

In tests with single-piece nozzles, the measured value of μ did not depend on the initial pressure in the vessel. For $N = 20$, a certain decrease in μ with the initial pressure was observed (see Table 3).

This is possibly due to the fact that the outflow was not rigorously adiabatic and the gas temperature increased on account of heat transfer from the walls. It is known [6] that the heat-transfer rate changes with pressure as $p^{0.8}$. The density thus varies inversely with T , and the sound velocity in the flow is in direct proportion to $T^{0.5}$. This reduces the flow rate and, hence, the discharge coefficient μ . Another possible cause of the change in μ with pressure is that the Reynolds number affects the drag coefficients of nozzles.

Conclusions.

1. A method for determining the gas-discharge coefficient for supersonic gas discharge from an orifice was developed.
2. It was shown that for supersonic gas discharge the discharge coefficients of combinations of nozzles in

the forward and reverse directions differ by a factor of 2. For a single-piece nozzle the ratio of the coefficients does not exceed 1.3.

3. For a combination of two nozzles, an optimum distance between the nozzles exists (about 1.5 diameters) for which the ratio of the discharge coefficients for the forward and reverse directions is maximum. The gas-mass flow for the reverse direction was found to be unstable for combinations of nozzles placed at this distance.

4. The greatest difference between the discharge coefficients for the forward and reverse directions was observed in the channels formed of nozzles with equal diameters. Combinations of nozzles with different diameters operate in much the same way as a single nozzle of the smallest diameter.

5. The discharge coefficient of combination of nozzles at a given gas temperature decreases as the initial pressure in the vessel is reduced.

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